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Particle decay in six-dimensional relativity

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Abstract. Special relativity is developed using a real six-dimensional space–time. It is shown that the imposition of parallel time displacements in each frame necessarily implies subluminal transformations. Conservation of six-momentum yields the usual three-momentum equation together with a vector equation which replaces the scalar energy equation of the corresponding four-dimensional theory. On energetic grounds alone it is shown that a particle may decay into a set of particles whose total rest mass is greater than that of the original particle, and that particles may be produced from the vacuum state. When virtual particle production is considered, this suggests a tentative link between the uncertainty principle and a six-dimensional theory with a restriction to small deviations from a constant time direction.

1. Introduction

When the special Lorentz transformations are extended to deal with superluminal reference frames it is found that imaginary quantities enter the linear transformations (Recami and Mignani 1974, Corbeń 1975, Pahor and Strnad 1976, Cole 1977, Recami 1978, 1979 *Rep. Prog. Phys.* (to be submitted)). A number of workers (Dorling 1970, Demers 1975, Kalitzin 1975, Mignani and Recami 1976, Ziino 1977, Cole 1978, Dattoli and Mignani 1978, Pappas 1978, Vyšín 1978) have investigated the idea that three time dimensions may be necessary for a fuller description of physical phenomena, mainly in connection with these extended transformations. Cole (1977) showed that, assuming the constancy of the speed of light, real linear tranformations could be obtained between six real space–time variables for both the subluminal and superluminal cases. In addition to retaining linearity and reality, this approach also has the advantage of introducing space–time symmetry when the two extra variables are interpreted as time coordinates.

The major problem associated with this approach is that of interpreting the idea of a three-dimensional time in terms of observations on physical systems. Time does not manifest itself to us as three-dimensional. It has been shown (Cole 1978) that the time dilation effect of special relativity may be recovered by averaging over all possible orientations of the three time axes, but this approach is unsatisfactory because it destroys the pleasing space–time symmetry and does not explain why observing processes produce averages over time but not space orientations. In the context of general relativity Cole (1979) has taken the metric

$$ds^2 = -a(r)\, dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\psi^2) + b(r)[(dt^1)^2 + (dt^2)^2 + (dt^3)^2] \quad (1.1)$$

for a static spherically symmetric situation, and solved the field equations exactly for $a(r)$ and $b(r)$. In the case of planetary motion, the resulting geodesic equations predict a perihelion advance of $\frac{7}{3}$ times that predicted by four-dimensional relativity, giving an unsatisfactory prediction of 100·4 seconds of arc per century in the case of Mercury.

The aim of this paper is to develop the six-dimensional formalism to a point at which particle decay may be discussed. The main result is that the six-momentum of a particle has the form $\begin{pmatrix} \boldsymbol{p} \\ \boldsymbol{E} \end{pmatrix}$ where $\boldsymbol{p}$ is its three-momentum and $\boldsymbol{E}$ is its vector energy directed along its time path. Conservation of six-momentum yields the usual three-momentum equation of the four-dimensional theory, while the single energy equation of that theory is replaced by a vector equation. When all time displacements are in the same direction then this vector equation reduces to the single equation. However, in general there is no conservation condition on the energy magnitudes alone, with the result that the theory allows decays of the type $e^- \to n + \bar{p} + \nu_e$, $p \to n + e^+ + \nu_e$, $p \to n + \pi^+$ and $p \to p + \pi^0$ to take place quite naturally in free space.

Clearly, on observational grounds, we are not allowed to consider any general time displacement because the isotropic substitution $dt^2 \to (dt^1)^2 + (dt^2)^2 + (dt^3)^2$ in (1.1) produces the wrong perihelion shift, and the proton and electron are stable. However, the last two virtual decay processes above are explained using the uncertainty principle. It is thus tentatively proposed that very slight deviations from a common time direction are allowed, the magnitude of these deviations being closely linked with $\hbar$. This would allow the production of virtual particles to occur and explain why time appears to be one-dimensional on a macroscopic level. No mechanism for the production of almost parallel time displacements is proposed, and the consequences of this tentative proposal are still under investigation.

Section 2 discusses the general six-dimensional special transformation, and § 3 shows how the usual four-dimensional theory emerges as a special case. Section 4 introduces the necessary six-vectors, and the principle of conservation of six-momentum is introduced in § 5. In § 6 it is proved that a particle of non-zero rest mass M_0 can decay into a set of particles whose total rest mass can be greater or less than M_0, and that particles of non-zero rest mass may be produced from the vaccum state. We take $c = 1$ throughout the paper.

2. Basic properties of the transformation

Let S and S' be two inertial frames with spatial origins O and O' respectively, and let unprimed and primed symbols refer to quantities measured in S and S' respectively. Let G be the 6×6 diagonal matrix

$$G = \begin{pmatrix} -\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix}$$

where $\mathbf{I}$ is the 3×3 identity matrix, and let $k = +1$ and -1 for subluminal and superluminal transformations respectively. Then for space and time increments $\mathbf{d}\boldsymbol{x}$ and $\mathbf{d}\boldsymbol{t}$ it has been shown (Cole 1978) that

$$\begin{pmatrix} \mathbf{d}\boldsymbol{x}' \\ \mathbf{d}\boldsymbol{t}' \end{pmatrix} = \Lambda \begin{pmatrix} \mathbf{d}\boldsymbol{x} \\ \mathbf{d}\boldsymbol{t} \end{pmatrix} \tag{2.1}$$

where Λ is a 6×6 matrix with constant coefficients such that $\Lambda^T G \Lambda = kG$. This result ensures that

$$|\mathbf{d}\boldsymbol{x}'|^2 - |\mathbf{d}\boldsymbol{t}'|^2 = k(|\mathbf{d}\boldsymbol{x}|^2 - |\mathbf{d}\boldsymbol{t}|^2). \tag{2.2}$$

Writing

$$\Lambda \equiv \begin{pmatrix} \mathbf{A} & \mathbf{P} \\ \mathbf{Q} & \mathbf{R} \end{pmatrix},$$

it follows that

$$\Lambda^{-1} = k\begin{pmatrix} \mathbf{A}^{\mathrm{T}} & -\mathbf{Q}^{\mathrm{T}} \\ -\mathbf{P}^{\mathrm{T}} & \mathbf{R}^{\mathrm{T}} \end{pmatrix} \tag{2.3}$$

and

$$\begin{aligned} &\mathbf{A}\mathbf{A}^{\mathrm{T}} - \mathbf{P}\mathbf{P}^{\mathrm{T}} = k\mathbf{I} \\ &\mathbf{A}^{\mathrm{T}}\mathbf{A} - \mathbf{Q}^{\mathrm{T}}\mathbf{Q} = k\mathbf{I} \\ &\mathbf{R}\mathbf{R}^{\mathrm{T}} - \mathbf{Q}\mathbf{Q}^{\mathrm{T}} = k\mathbf{I} \\ &\mathbf{R}^{\mathrm{T}}\mathbf{R} - \mathbf{P}^{\mathrm{T}}\mathbf{P} = k\mathbf{I} \\ &\mathbf{A}\mathbf{Q}^{\mathrm{T}} = \mathbf{P}\mathbf{R}^{\mathrm{T}} \\ &\mathbf{A}^{\mathrm{T}}\mathbf{P} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}. \end{aligned} \tag{2.4}$$

Further, if $k = +1$ then $|\mathbf{A}| \neq 0$ and $|\mathbf{R}| \neq 0$; if $k = -1$ then $|\mathbf{P}| \neq 0$ and $|\mathbf{Q}| \neq 0$. The work that follows has not been reported elsewhere.

The motion of a particle may be specified in a frame by its velocity $\boldsymbol{v} = \mathbf{d}\boldsymbol{x}/\mathrm{d}t$ where $\mathrm{d}t^2 = |\mathbf{d}\boldsymbol{t}|^2 = (\mathrm{d}t^1)^2 + (\mathrm{d}t^2)^2 + (\mathrm{d}t^3)^2$, and by the unit vector $\boldsymbol{\alpha} = \mathbf{d}\boldsymbol{t}/\mathrm{d}t$ which represents the direction of its time displacement. This specification may be denoted by the six-component column vector $\begin{pmatrix} \boldsymbol{v} \\ \boldsymbol{\alpha} \end{pmatrix}$. Let the motion of O be specified by the vectors $\begin{pmatrix} \boldsymbol{O} \\ \boldsymbol{\alpha}_O \end{pmatrix}$ and $\begin{pmatrix} \boldsymbol{v}' \\ \boldsymbol{\alpha}'_O \end{pmatrix}$ in S and S' respectively, and let the motion of O' be specified by the vectors $\begin{pmatrix} \boldsymbol{v} \\ \boldsymbol{\alpha}_{O'} \end{pmatrix}$ and $\begin{pmatrix} \boldsymbol{O} \\ \boldsymbol{\alpha}'_{O'} \end{pmatrix}$ in S and S' respectively. Now let an infinitesimal increment along the world line of O have displacements $\begin{pmatrix} \boldsymbol{O} \\ \boldsymbol{\alpha}_O\,\mathrm{d}t_O \end{pmatrix}$ in S and $\begin{pmatrix} \boldsymbol{v}'\,\mathrm{d}t'_O \\ \boldsymbol{\alpha}'_O\,\mathrm{d}t'_O \end{pmatrix}$ in S', and let an infinitesimal increment along the world line of O' have displacement $\begin{pmatrix} \boldsymbol{v}\,\mathrm{d}t_{O'} \\ \boldsymbol{\alpha}_{O'}\,\mathrm{d}t_{O'} \end{pmatrix}$ in S and $\begin{pmatrix} \boldsymbol{O} \\ \boldsymbol{\alpha}'_{O'}\,\mathrm{d}t'_{O'} \end{pmatrix}$ in S'. (In this paper, all vectors $\boldsymbol{\alpha}$, with and without primes or subscripts, denote unit vectors.) Substituting these two schemes in turn into (2.1) and its inverse using (2.3) and defining $\gamma \equiv \mathrm{d}t_{O'}/\mathrm{d}t'_{O'}$ and $\gamma' \equiv \mathrm{d}t'_O/\mathrm{d}t_O$, one finds

$$\begin{aligned} &\mathbf{A}\boldsymbol{v} + \mathbf{P}\boldsymbol{\alpha}_{O'} = \boldsymbol{O} \\ &\mathbf{A}^{\mathrm{T}}\boldsymbol{v}' - \mathbf{Q}^{\mathrm{T}}\boldsymbol{\alpha}'_O = \boldsymbol{O} \\ &\boldsymbol{\alpha}'_{O'} = \gamma(\mathbf{Q}\boldsymbol{v} + \mathbf{R}\boldsymbol{\alpha}_{O'}) \\ &\gamma\boldsymbol{v} = -k\mathbf{Q}^{\mathrm{T}}\boldsymbol{\alpha}'_{O'} \\ &\gamma\boldsymbol{\alpha}_{O'} = k\mathbf{R}^{\mathrm{T}}\boldsymbol{\alpha}'_{O'} \\ &k\boldsymbol{\alpha}_O = \gamma'(-\mathbf{P}^{\mathrm{T}}\boldsymbol{v}' + \mathbf{R}^{\mathrm{T}}\boldsymbol{\alpha}'_O) \\ &\gamma'\boldsymbol{v}' = \mathbf{P}\boldsymbol{\alpha}_O \\ &\gamma'\boldsymbol{\alpha}'_O = \mathbf{R}\boldsymbol{\alpha}_O. \end{aligned} \tag{2.5}$$

The matrices **A**, **P**, **Q** and **R** are thus given in terms of the velocity vectors $\boldsymbol{v}$ and $\boldsymbol{v}'$ and the time displacement vectors $\boldsymbol{\alpha}_O$, $\boldsymbol{\alpha}'_O$, $\boldsymbol{\alpha}_{O'}$ and $\boldsymbol{\alpha}'_{O'}$ by relations (2.4) and (2.5). These can be combined to give

$$\gamma^2 = k(1-v^2)^{-1}, \qquad \gamma'^2 = k(1-v'^2)^{-1}$$

and

$$\gamma\boldsymbol{\alpha}_{O'} \cdot \boldsymbol{\alpha}_O = k\gamma'\boldsymbol{\alpha}'_{O'} \cdot \boldsymbol{\alpha}'_O.$$

Thus for real transformations, $k = +1$ if both $|\boldsymbol{v}|$ and $|\boldsymbol{v}'|$ are less than 1, and $k = -1$ if both $|\boldsymbol{v}|$ and $|\boldsymbol{v}'|$ are greater than 1. This result is consistent with the classification of subluminal and superluminal velocities.

3. Link with four-dimensional relativity

Now assume that in each frame all time displacements are in the same direction. On writing $\boldsymbol{\alpha}_O = \boldsymbol{\alpha}_{O'} \equiv \boldsymbol{\alpha}$ and $\boldsymbol{\alpha}'_O = \boldsymbol{\alpha}'_{O'} \equiv \boldsymbol{\alpha}'$, the results of the last section give $\gamma' = k\gamma$, $v'^2 = v^2$ and $\mathbf{R}\boldsymbol{\alpha} = k\gamma\boldsymbol{\alpha}'$. Then transformation (2.1) with $\mathbf{d}\boldsymbol{t}' = \mathrm{d}t'\,\boldsymbol{\alpha}'$ and $\mathbf{d}\boldsymbol{t} = \mathrm{d}t\,\boldsymbol{\alpha}$ gives $\mathbf{Q}\,\mathbf{d}\boldsymbol{x} = (\mathrm{d}t' - k\gamma\,\mathrm{d}t)\boldsymbol{\alpha}'$ for each displacement $\mathbf{d}\boldsymbol{x}$. It follows that $\mathbf{Q}$ is such that for each vector $\boldsymbol{\beta}$ there exists a number μ such that $\mathbf{Q}\boldsymbol{\beta} = \mu\boldsymbol{\alpha}'$, which implies that $|\mathbf{Q}| = 0$ and $k = +1$. Thus the imposition of parallel time directions in each frame implies subluminal transformations.

4. Six-vectors

Covariant and contravariant six-vectors A_i and B^i transform according to $A'_i = (\partial x^j/\partial x'^i)A_j$ and $B'^i = (\partial x'^i/\partial x^j)A^j$, with the usual extension to higher-order tensors. Raising and lowering indices can be defined using the metric six-tensor in the usual way, and the inner product is preserved. In particular, the displacement $\mathrm{d}x = \begin{pmatrix}\mathbf{d}\boldsymbol{x}\\ \mathbf{d}\boldsymbol{t}\end{pmatrix}$ in six-dimensional space–time is a six-vector.

Consider a frame S in which a particle moves instantaneously with three-velocity $\boldsymbol{v}$ and time displacement $\boldsymbol{\alpha}$, and let S' be an instantaneous rest frame of the particle which is situated at O'. Using (2.2) it is clear that $\mathrm{d}t'_{O'}$ is independent of the orientation of the time axes of S', so that $U \equiv \mathrm{d}x/\mathrm{d}t'_{O'}$ is a contravariant six-vector, called the six-velocity. Then

$$U = \frac{\mathrm{d}t_{O'}}{\mathrm{d}t'_{O'}}\frac{\mathrm{d}x}{\mathrm{d}t_{O'}} = \gamma\begin{pmatrix}\boldsymbol{v}\\ \boldsymbol{\alpha}\end{pmatrix}.$$

The six-acceleration is defined as

$$A \equiv \mathrm{d}U/\mathrm{d}t'_{O'} = \gamma\,\mathrm{d}U/\mathrm{d}t_{O'},$$

the six-momentum is defined as

$$P \equiv m_0 U = m_0\gamma\begin{pmatrix}\boldsymbol{v}\\ \boldsymbol{\alpha}\end{pmatrix}$$

where m_0 is the rest mass of the particle, and the six-force is defined as

$$F \equiv \mathrm{d}P/\mathrm{d}t'_{O'} = m_0 A.$$

Then the result $G_{\mu\nu}\,\mathrm{d}x^{\mu}\,\mathrm{d}x^{\nu} = k(\mathrm{d}t'_{O'})^2$ implies that $U_\nu U^\nu = k$, which in turn implies that $F_\mu U^\mu = m_0 A_\mu U^\mu = 0$.

5. Conservation of six-momentum

Consider a set of particles $i = 1, \ldots, N$ such that the ith particle has velocity $\boldsymbol{v}_i$, time vector $\boldsymbol{\alpha}_i$ and mass m_i in frame S. Its six-velocity in S will be

$$U_{(i)} = \gamma(v_i)\begin{pmatrix}\boldsymbol{v}_i\\ \boldsymbol{\alpha}_i\end{pmatrix}$$

where $\gamma(v_i) = [k_i(1-v_i^2)^{-1}]^{1/2}$ and $k_i = \pm 1$. The following result then holds: if the law

$$\sum_{i=1}^{N} m_i \boldsymbol{v}_i = \boldsymbol{\pi} = \textbf{constant}$$

and (5.1)

$$\sum_{i=1}^{N} m_i \boldsymbol{\alpha}_i = \boldsymbol{\mu} = \textbf{constant}$$

holds in frame S then it holds in any other inertial frame if

$$m_i = m_{i0}\gamma(v_i) \qquad (i = 1, \ldots, N) \tag{5.2}$$

where the m_{i0} are frame-independent scalars.

This result is proved as follows: using (5.2), equations (5.1) can be written

$$\sum_{i=1}^{N} m_{i0} U_{(i)} = \begin{pmatrix}\boldsymbol{\pi}\\ \boldsymbol{\mu}\end{pmatrix} = \text{constant in } S.$$

Then

$$\sum_{i=1}^{N} m_{i0} U'^{\nu}_{(i)} = \sum_{i=1}^{N} m_{i0} \frac{\partial x'^{\nu}}{\partial x^{\rho}} U^{\rho}_{(i)}$$

is constant in S' for $\nu = 1, \ldots, 6$, so that (5.1) holds in S'.

For subluminal particles, (5.2) is the usual velocity dependence of the mass, with m_{i0} being the rest mass of the ith particle. For superluminal particles the mass decreases from infinity at $|\boldsymbol{v}| = 1$ to the value zero as $|\boldsymbol{v}| \to \infty$. Law (5.1) corresponds to the conservation of six-momentum of the system.

For a single particle of mass $m = \gamma(v)m_0$ in frame S, define the vector $\boldsymbol{E} = m\boldsymbol{\alpha}$ with magnitude $E = |\boldsymbol{E}| = m$. The six-momentum of the particle is then

$$P = \begin{pmatrix} m\boldsymbol{v}\\ m\boldsymbol{\alpha}\end{pmatrix} = \begin{pmatrix}\boldsymbol{p}\\ \boldsymbol{E}\end{pmatrix}$$

where $\boldsymbol{p} = m\boldsymbol{v}$ is the three-momentum in S. It follows from (5.2) that $E^2 = p^2 + km_0^2$. The six-force on the particle is then

$$F = \gamma \frac{\mathrm{d}}{\mathrm{d}t_{O'}}\begin{pmatrix}\boldsymbol{p}\\ E\boldsymbol{\alpha}\end{pmatrix} = \gamma\begin{pmatrix}\boldsymbol{f}\\ \mathrm{d}(E\boldsymbol{\alpha})/\mathrm{d}t_{O'}\end{pmatrix}$$

where $f \equiv \mathrm{d}p/\mathrm{d}t_{O'}$ is the three-force in S. Thus

$$0 = F_\mu U^\mu = \gamma^2\left(\alpha \cdot \frac{\mathrm{d}}{\mathrm{d}t_{O'}}(E\alpha) - f \cdot v\right)$$

which gives $\mathrm{d}E = f \cdot v \, \mathrm{d}t_{O'} = f \cdot \mathrm{d}x$. Thus $\boldsymbol{E}$ is interpreted as a vector energy.

6. Particle decay

In this section we consider only non-tachyon particles. A particle with rest mass M_0, time vector α and zero three-momentum in the laboratory frame decays into n particles. The ith particle has rest mass M_i, time vector α_i and momentum p_i in this frame. The following results hold.

(i) If $M_0 = 0$ (particle production from the vacuum state) then production is possible for any values of the M_i provided $n \geqslant 3$, while production is possible for $n = 2$ only if $M_1 = M_2$.

(ii) If $M_0 \neq 0$ then decay is possible for any values of the M_i provided $n \geqslant 2$.

The proofs are as follows: conservation of six-momentum with $k_i = 1$ gives

$$M_0\alpha = \sum_{i=1}^{n} E_i\alpha_i = \sum_{i=1}^{n} (M_i^2 + p_i^2)^{1/2}\alpha_i$$

and

$$\mathbf{0} = \sum_{i=1}^{n} p_i.$$

Consider the particular decay $p_1^2 = p_2^2 = \ldots = p_n^2 \equiv p^2$. Then

$$M_0\alpha = \sum_{i=1}^{n} (M_i^2 + p^2)^{1/2}\alpha_i.$$

In geometrical terms we only need to find a value of p such that an $(n+1)$-sided closed circuit can be found with sides of lengths M_0 and $(M_i^2 + p^2)^{1/2} (i = 1, \ldots, n)$. If $M_0 = 0$ we may take $p^2 = \Sigma_{i=1}^n M_i^2$ provided $n \geqslant 3$, while if $M_0 \neq 0$ we may take $p^2 = M_0^{-2} \Sigma_{i=0}^n M_i^4$, for it is easily verified in each case that the sum of the lengths of any n edges is not less than the length of the remaining edge. Thus for arbitrary values of the M_i, particular decays have been found. For the case $M = 0$ and $n = 2$ we must have $p_1 = p_2$, $\alpha_1 = -\alpha_2$ and $M_1 = M_2$ (arbitrary). This completes the proof.

Thus on energetic grounds, the processes $e^- \to n + \bar{p} + \nu_e$, $e^+ \to K^+ + \bar{\nu}_e$, $p \to n + e^+ + \nu_e$, $p \to p + \pi^0$ and $p \to n + \pi^+$ are allowed in free space in the framework of six-dimensional special relativity, but not in the four-dimensional framework. The first decay predicts electron instability while the last three processes predict proton instability. The third process is predicted in the four-dimensional theory, but only in the presence of a nucleus. Proton instability is also predicted in SU(5) with a lifetime of 10^{33} yr (Close 1979 and references therein).

Thus, in conclusion, observational evidence on planetary orbits and electron–proton stability points to the fact that, if time is more than one-dimensional, then it is highly directed in some way. One possible clue emerges: the last two decay processes can be accounted for by invoking the uncertainty principle $\Delta E \, \Delta t \geqslant \hbar$. Thus it may be that time displacements are strongly directed along a common direction, with very small deviations from this direction manifesting themselves only on a microscopic scale. The magnitude of these deviations will depend in some way on the value of $\hbar$. Such a theory would allow a prediction of the uncertainty principle by considering the geometry of six-dimensional space–time with a restriction to very small deviations from a common time direction. This idea is under current investigation.

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